A Warming Arctic Would Not Cause Increased Severe Weather or Temperature Extremes

by Chuck Wiese, Meteorologist, Weatherwise, Inc.

This paper is a critique Francis and Vavrus (2012), hereinafter FV (2012), by atmospheric scientists Jennifer Francis from Rutgers University and Steve Vavrus of the University of Wisconsin. Their paper can be downloaded here and an updated version here:

FV (2012) claims a measured decrease in the zonal or west to east wind component due to "arctic amplification" (AA) would increase jet stream meandering, increase the amplitude or "waviness" of the flow, and increase persistent long wave blocking patterns around the northern hemisphere. This, in turn, would increase severe weather, droughts, floods and temperature extremes.

To quote the authors directly:

> *"Two effects are identified that each contribute to a slower eastward progression of Rossby waves in the upper-level flow: 1) weakened zonal winds, and 2) increased wave amplitude. These effects are particularly evident in autumn and winter consistent with sea-ice loss, but are also apparent in summer, possibly related to earlier snow melt on high-latitude land. Slower progression of upper-level waves would cause associated weather patterns in mid-latitudes to be more persistent, which may lead to an increased probability of extreme weather events that result from prolonged conditions, such as drought, flooding, cold spells, and heat waves."*

To quote the authors again, the effects described above are the result of arctic amplification, a term defined by the authors as:

> *"Arctic amplification (AA) – the observed enhanced warming in high northern latitudes relative to the northern hemisphere"*

This definition seems to fit the claims made by NASA GISS and NOAA that temperature measurements of the arctic are warming at a much greater rate than anywhere else in the northern hemisphere.

To examine these claims by the authors, I will use an application of dynamic meteorology from atmospheric science and introduce the physics of Rossby waves, invoked by the authors as applicable to validating their claims as well as a few of the governing laws of motion that describe the behavior of these waves and how they would interact with a warming arctic.

The physics of wave motion can become a very math intensive discussion. For the sake of article simplification, I will provide the steps of deriving these governing equations which used to be used in synoptic meteorology as a tool by forecast meteorologists in appendix 1 with use of the resulting equations to estimate how a particular wave pattern in the atmosphere will behave in a generalized sense.

This was before modern computing power enabled meteorologists to expand on these same ideas and obtain through that computing power a complex and comprehensive set of equations that can deal with each hemispheric weather system and its associated Rossby wave individually and affect the answers of all in greater specificity. BUT this does not affect the purpose or conclusion derived in writing this article whatsoever.

Two terms come into the discussion which are important to define. They are the amplitude and length of a wave, called amplitude and wavelength respectively. In the diagrams below, the amplitude of a wave is the amount of northward and southward stretching it can assume in time, which would be along the *y*-axis in Cartesian coordinates, paralleling the north and south component of the wind, *v*. It is a measure of the *y* or north-south distance between a ridge and trough axis.

A high amplitude wave or flow means a greater *y* distance between the ridge and trough peaks. These are called long waves in the westerlies and are considered full latitude waves often starting as low as 30 degrees north latitude and extending as high as 80 degrees north latitude. The wavelength is defined as the lateral spacing between the waves over 2 *pi* radians since the waves are of a trigonometric form and are measured along the *x*-axis or west-to-east direction in Cartesian coordinates, paralleling the west to east wind component, *u*, referenced by FV (2012) as the "*zonal wind flow*".

Figure 1: Wave Amplitude

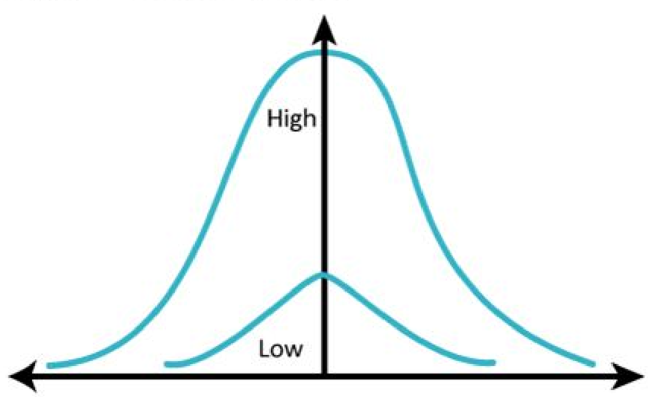
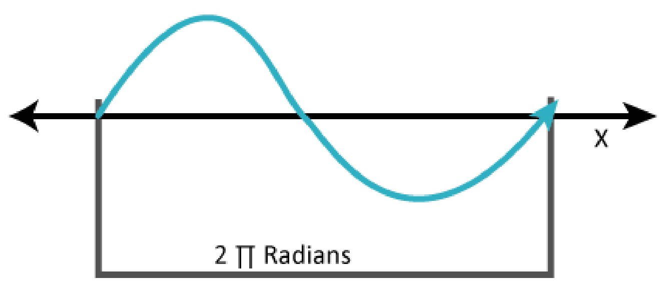


Figure 2: Wavelength



FV (2012) claims Rossby wave physics shows *arctic amplification* (AA) and weakened horizontal temperature gradient will decrease the westerly or zonal wind component in the Arctic. This decrease causes atmospheric waves to increase in number, amplify and stall over particular regions of the earth. This stalling increases severe weather, floods, and low-temperature extremes under the troughs, and droughts with high-temperature extremes under the ridges.

To examine FV (2012)'s claim, we need to define Rossby waves and look at how AA might change how Rossby waves behave. In the Appendix 1, we derive and explain the Rossby equation. Here, we will use the Rossby equation to demonstrate the first flawed assumption in FV (2012).

In Appendix 2, Eq (11) we derive,

$$U - c = \beta L^2 / 4\pi^2 \qquad (11)$$

where

U = the zonal and hemispheric speed of the west winds

c = the speed of the individual waves traveling within the flow

beta = the Rossby parameter

L = the length between the waves spanning 2 *pi* radians

The terms that arrive from the general solution for *v'* in the Appendix 1.

This is the Rossby equation, derived in 1939 by Carl Rossby, who became a famous scientist for his work in atmospheric dynamics. It is immediately apparent that based upon the computation of the mean zonal speed of the westerlies across the hemispheres, that a good idea can be surmised as to how many planetary waves we could expect to set up based upon the use of this equation.

For easterly moving waves which have a positive value of *c*,

$(U - c) < U$

because $U > c$ and $c > 0$.

For westerly moving waves which have a negative value of *c*,

$(U - c) > U$

because $U > c$ and $c < 0$, and the length of the waves would be relatively long instead of relatively short.

In other words, the higher the wave speed compared to the zonal current, the shorter the wave lengths.

As the waves slow in progression, we see that values of $(U - c)$ approach *U* and we get much longer wavelengths for a given value of *U*, hence less planetary waves around the hemispheres. The longest wave lengths are apparent when the waves actually retrogress in the atmosphere, in other words, move from east to west. In such a manner, we then have a negative number for *c* and it is obvious the value of $(U - c)$ becomes greater than *U* for the maximum wavelength permissible.

In terms of the speed of the waves, the Rossby equation demonstrates that FV (2012) is incorrect, because as the waves slow with respect to the zonal winds, the wave lengths increase, which is the opposite of what they claim because that result decreases the waviness of the flow around the hemispheres. But what the authors also argue to claim more waviness to the jet stream (counter-intuitive to the wave speed) is that the speed of the current *U* or jet stream speed is declining due to Arctic amplification weakening the horizontal gradient of temperature across the latitude lines.

To examine this, we will further simplify the Rossby equation by assuming the persistent scenario claimed by FV (2012) of stagnating weather patterns giving weather extremes results from the waves becoming stationary or standing in the flow which they occasionally do. We then set $c = 0$ for wave speed and solve the Rossby equation for *L*. The result is we get an arbitrary maximum wavelength L_x that depends on the zonal jet stream speed and latitude from the Rossby parameter,

$$L_x = 2\pi\sqrt{U/\beta}$$

While it is apparent from this equation that the length between the waves can decrease from a declining zonal wind speed *U* as claimed (a more wavy flow) it is also apparent that based upon the measured decline of the westerly winds in the data estimated by the authors (about a 14% decline) that this does not increase the number of waves with any weather significance around the hemisphere if one considers that impact over the range of speeds we find in the real atmosphere. The most common pressure level that is closest to a nondivergent level required for use in the Rossby equation is near 600 millibars or just under 18,000 ft. The atmospheric range there is between 10 to 50 m/s.

Using this equation, I prepared a table below (Figure 3) which shows us the number of permissible standing waves based upon the speed of the zonal current at latitude 45 degrees North and 60 degrees North over the normal range of speed found and then with a 14% reduction. As we can see, it makes little difference and the number of waves or "waviness" in the flow remains nearly the same for most of the westerly wind speeds. So again, the authors are incorrect according to the Rossby physics.

Figure 3

60 ° North Latitude

Number of Waves Hemispheric	6	4	3	2	2
U ms^{-1}	10	20	30	40	50
Number of Waves Hemispheric	6	4	3	3	2
U ms^{-1} 14% Reduction	8.6	17.2	25.8	34.4	43

45 ° North Latitude

Number of Waves Hemispheric	6	4	3	3	3
U ms^{-1}	10	20	30	40	50
Number of Waves Hemispheric	6	4	4	3	3
U ms^{-1} 14% Reduction	8.6	17.2	25.8	34.4	43

Critics of the use of this equation will try and argue that it is over-simplified in explaining the behavior of a chaotic system such as the earth's atmosphere and that wavelengths are much more unstable and non-uniform in nature, unlike the limitation the mathematics places on the use of this equation. While part of that is true, it is also true that we do not use this equation alone to describe wave behavior in the atmosphere in an absolute sense.

Phasing equations and other physical equations determining the more complex motions of the atmosphere not related entirely to Rossby physics are used in weather models to incorporate the more complete set of governing behavior. BUT it is absolutely true that in a climatic sense such as what is claimed in FV (2012), this equation derived by Rossby gives us an accurate portrayal of how the waves around the hemispheres would behave and change in a general sense if their claims were true, for this equation is an important building block that sets the foundation for all of the other wave behavior and we see clearly in this part of the article that the waves would not behave as the authors claim in their peer reviewed paper.

The lateral spacing of the earth's planetary waves or wavelengths is only half of the problem at hand to check on the validity of the claims in FV (2012).

The second and just as critical component is examining how the amplitude of these waves would change if the westerly winds across the northern hemisphere began to slow because of AA or Arctic Amplification warming as the authors claim is happening, which according to their claims would cause the amplitude of the waves to increase.

Platzman (1947) derived an expression as an aid to solving for wave amplitude by deriving a trigonometric form that allows for the conversion of the earth's geometry in lateral spacing along latitude circles to project onto a mapping in Lambert conformal or stereographic coordinates. That can then be plugged into the Rossby physics as we show in Appendix 1 to derive an expression in spherical coordinates that can be solved for the amplitude of the Rossby waves.

In Appendix 3, we derive,

$$v_0 = y - y_0 = 2\sqrt{V/\beta[(cos\psi/sec\rho cos\rho_1) - cos\psi_1)]} \qquad (19)$$

Immediately, we can see a remarkable similarity of this expression versus the Rossby equation that describes the length of the waves. If we set aside the trigonometric terms for a moment, we see that in a standing wave format described above by L_x the only difference is we are not summing π radians but just twice the quantity of the square root of V/β multiplied by the trigonometric terms that size the amplitude.

Then V is substituted for U meaning that instead of a prior prescribed and mean or steady westerly wind belt, we now have a total and point specific wind velocity streamline vector whose wave amplitude is dependent not only on the magnitude of V but on the DIRECTION of the wind vector that is described by the variables ψ and ψ_1. The subscript "1" refers to the wind direction at what we call the inflection point latitude of the waves, or when the wind streamlines make a clear break in direction from a trough to ridge axis.

Likewise, ρ and ρ_1 are the maximum amplitude latitude and inflection point latitudes respectively of the waves which are typically found to be roughly one half of the amplitude of the waves from trough to ridge axis, as a rule of thumb. Now the variable v_0 or amplitude no longer has to be prescribed as a constant as it was in the case of the equation that described Rossby wavelengths, but varies with wind speed, direction, and latitude, all prescribed in the above equation.

If we compare the physical meaning of what defines Rossby wavelength and speed in these equations, the corollary is clear. As the wind speed increases along the given waves, so must the amplitude and wavelength. As speed decreases, so must the amplitude of the waves and their respective wavelengths, with the maximum amplitude of the waves being realized when the inflectional direction has a backed wind direction to south or even east in the case of closed off low pressure streamlines. THIS IS CLEARLY IN CONTRADICTION AND OPPOSITE OF WHAT IS CLAIMED BY FV (2012).

To illustrate this point, let us take a case of demonstrating what happens to the wind direction and speed of the jet stream when it is exposed to increasing gradients of temperature, or the opposite effect of what FV (2012) claims is happening in their paper.

Have you ever wandered outside, looked up into the sky and felt the wind blowing against your back or face and then when looking up at passing clouds that they are moving from a totally different direction from the wind that blows against your face or back?

This happens frequently in the real world and is a visual example of what happens to the winds going upward in the vertical when the wind and its associated pressure surfaces no longer parallel the isotherms. When this happens, the atmosphere leaves the state of a more energetic stability and increases and liberates potential energy as the wind begins to blow more cross isothermal or at increasingly normal angles to the isotherms.

When this happens, the wind is now moving respective warm and cold air masses to different latitudes and longitudes and with the help of Rossby wave behavior, also allows the temperature difference over a fixed amount of space (the temperature gradient) to begin to increase. That process creates weather frontal systems that begin to generate lift and start the process of creating a low-pressure system or storm.

In the illustrations below, we start with a developing low-pressure system so that along the stacking lines of temperature gradient we have a cold front defined by those isotherms and the gradient of temperature is taken as 2 deg C across one hundred miles of latitude. The surface wind is completely normal to or at a 90-degree angle to the isotherms as the surface wind barb indicates 30 mph of wind from the west. The isotherms are oriented north to south for a west to east temperature gradient.

As the low-pressure circulation begins development, the isotherms rotate 90 degrees from east to west to north and south as depicted with the wind remaining normal to the isotherms, thus pushing them eastward.

So, what does this process do to the vertical wind profile?

We find that as storms develop, the vertical wind profile will shear with height, either backing from the surface wind direction (counterclockwise) or veer clockwise which depends on whether a colder or warmer air mass is headed towards you, respectively. But the speed of the wind also changes or shears with height, increasing as you go upward, and that is what we are interested in determining as the temperature gradients increase or decrease along such a system so that we can plug the results into the Rossby amplitude expression and see how the increasing or decreasing wind speed changes the wave amplitude.

In the examples below, the isotherms are oriented north to south so we want the portion of the thermal wind equation that represents how the thermal wind which parallels the isotherms will back the surface wind and increase it with height if we maintain the temperature gradient of 2 deg C through 500 millibars or about 18,000 ft of pressure altitude. The expression we need from Hess (1959) is

$$\Delta v \approx (g/fT)\frac{\partial T}{\partial x}\Delta z$$

Where the derivatives of *v* and *z* are taken as finite increments as Δ*v* is then the change in the northward component of the wind for a fixed increment of geopotential height, Δ*z*.

The letter *g* is the earth's gravity, *f* the Coriolis parameter already defined, *T* is the mean temperature of the layer of Δ*z* we are considering, which is 18,000 ft deep and the partial derivative of *T* with respect to *x* or the gradient of temperature on the west to east axis the isotherms are plotted on. This is also a finite quantity since it now has no other dependent variables.

The mean temperature of this layer is from a US standard atmosphere and therefore has a value of 270.25 deg K. Plugging in the relative numbers we have,

$$\Delta v \approx (1.2178 \exp -4 deg/s^2/3.0192 \exp -2 deg/s)(5,486.4m) = 22.12ms^-1$$

The west wind at the surface is given as 30 mph so we convert to the like unit of meters per second and get $13.41 m/s. As stated, the thermal wind is a shear vector whose top is added to the bottom value to get the resulting wind at the top so

$$tan\theta = v/u = 22.12ms^-1/13.41ms^-1 = 1.65, \theta = 59^0$$

Then the resulting wind vector at 18,000 ft has backed 59° from west to 270° – 59° = 211° or to the southwest at $211°. The magnitude of this wind vector is the square root of the sum of the squares of the west and south wind components respectively, therefore,

$$V = \sqrt{u^2 + v^2} = \sqrt{179.83m^2/s^2 + 489.29m^2/s^2} = 25.86ms^-1 = 58mph$$

So now we have a cold front and low-pressure system whose temperature gradient through 18,000 ft of geopotential height maintains 2 deg C of temperature gradient across the front over 100 statute miles and results in the west surface wind direction and speed of 30 mph backing at 18,000 ft to a direction of 211° true at 58 mph.

With this result, notice that we now have the wind vector *V*, so that we can go back to the Rossby amplitude expression to check on what amplitude this developing storm would likely generate.

The variables are the ridge axis wind direction and latitude, ψ and ρ and ψ_1 and ρ_1 (defining the wave amplitude from the inflection point) respectively for the cosine and secant.

In Fig 4 below, we must have a west wind direction, ψ = 0, for the ridge axis and the ridge axis latitude is 61.5^o degrees north or ρ = 61.5^o which we determine from the following steps.

At the inflection latitude, we have the variables ρ_1 and ψ_1 which are the inflectional latitude and wind direction. Taking those from the above for cosines we have ρ_1 = 50^o and ψ_1 = 59^o.

The wind vector magnitude was calculated to be 58 mph or 25.86 m/s. The *beta* for the Rossby parameter is 1.471 $\exp^{-11}$ m^{-1} s^{-1} for 50 degrees north latitude. With the inflectional wind direction, speed and latitude, you then pick an arbitrary "first estimate" latitude for the maximum wave amplitude defined at the ridge axis with a west wind for ρ.

We need to match the prediction of this maximum amplitude with the true latitudinal distance from ρ_1 to ρ, keeping in mind that what the equation is doing is integrating the effect of Coriolis turning across the latitude lines from the Rossby parameter contained within.

Therefore, if our first estimate is too small a latitude change or amplitude, the equation will predict too large an amplitude compared to the true distance between latitudes selected. On the other hand, if our first estimate exceeds the true wave amplitude, the equation will predict an amplitude too small compared to the true distance between the estimated maximum amplitude latitude and the inflection latitude. The inflection latitude was arbitrarily chosen.

After narrowing the prediction estimates to match near the true latitudinal distance, we are able to correctly calculate the maximum Rossby amplitude from the inflection point. This gives,

$$v_0 = 2\sqrt{(25.86ms^-1/1.471\exp -11m^-1s^-1)(.7425 - .5150)}$$
$$v_0 = 2(6.3310\exp 5m)$$
$$v_0 = 1.26620\exp 6m/1609msmile^-1 = 787miles$$

The maximum amplitude of this wave is 787 statute miles north of the inflection point to ridge axis latitude as depicted in Fig 4 below.

A verification of this answer is confirmed by cross checking it with CAVT trajectories, calculated from inflectional data by Hess & Fomenko (1955).

Note that we are not attempting nor does this equation predict the meridional displacement of the wave. For that can be easily referenced from the tables and we find from this that the amplitude peak is displaced 36 degrees of longitude east of 130 degrees west or at 94 degrees west longitude.

According to FV (2012), as the zonal winds decrease, we are supposed to see slowing progression of the waves with the amplitudes of the waves INCREASING as the zonal winds decrease from AA or Arctic Amplification warming BECAUSE OF THE DECREASING GRADIENT OF TEMPERATURE.

So in the first example given in Fig 4, we have an amplitude from the inflection point of 787 statute miles and the upper-level wind speed at 18,000 ft of *V* = 58 mph. This results from a temperature gradient of 2 deg C per 100 miles of west to east distance as depicted and maintaining that gradient through 18,000 ft of geopotential height.

So if we increase the gradient of temperature in Fig 4 by 2.5 times 2 deg C to 5 deg C per 100 miles of west to east distance along the same front with the same west wind direction increased to 40 mph at the inflection latitude, we can repeat the calculations and check the answer for amplitude.

The result is that because the gradient of temperature is increased by a factor of 2.5 the wind vector *V* at 18,000 ft now increases to 130 mph and backs yet further to 72 degrees from west or 198 degrees true at 130 mph as depicted in Fig 5. This result if FV (2012) is correct should give us a lower amplitude wave value compared to Fig 4.

Repeating the calculating steps from Fig 4 for Fig 5 gives

$$v_0 = 2\sqrt{(58.16ms^-1/1.471\exp -11m^-1s^-1)(.5703 - .3090)}$$
$$v_0 = 2(1.016425m\exp 6)$$
$$v_0 = 2.03285m\exp 6/1609msmile^-1 = 1,263miles$$

The maximum amplitude of this wave is 1,263 statute miles from the inflection point latitude. This is 60.5% greater than with the weaker temperature gradient depicted in figure 4, the exact opposite of what is claimed will happen in FV (2012) from decreasing winds due to decreasing temperature gradient from AA or Arctic Amplification warming. FV (2012) is clearly incorrect.

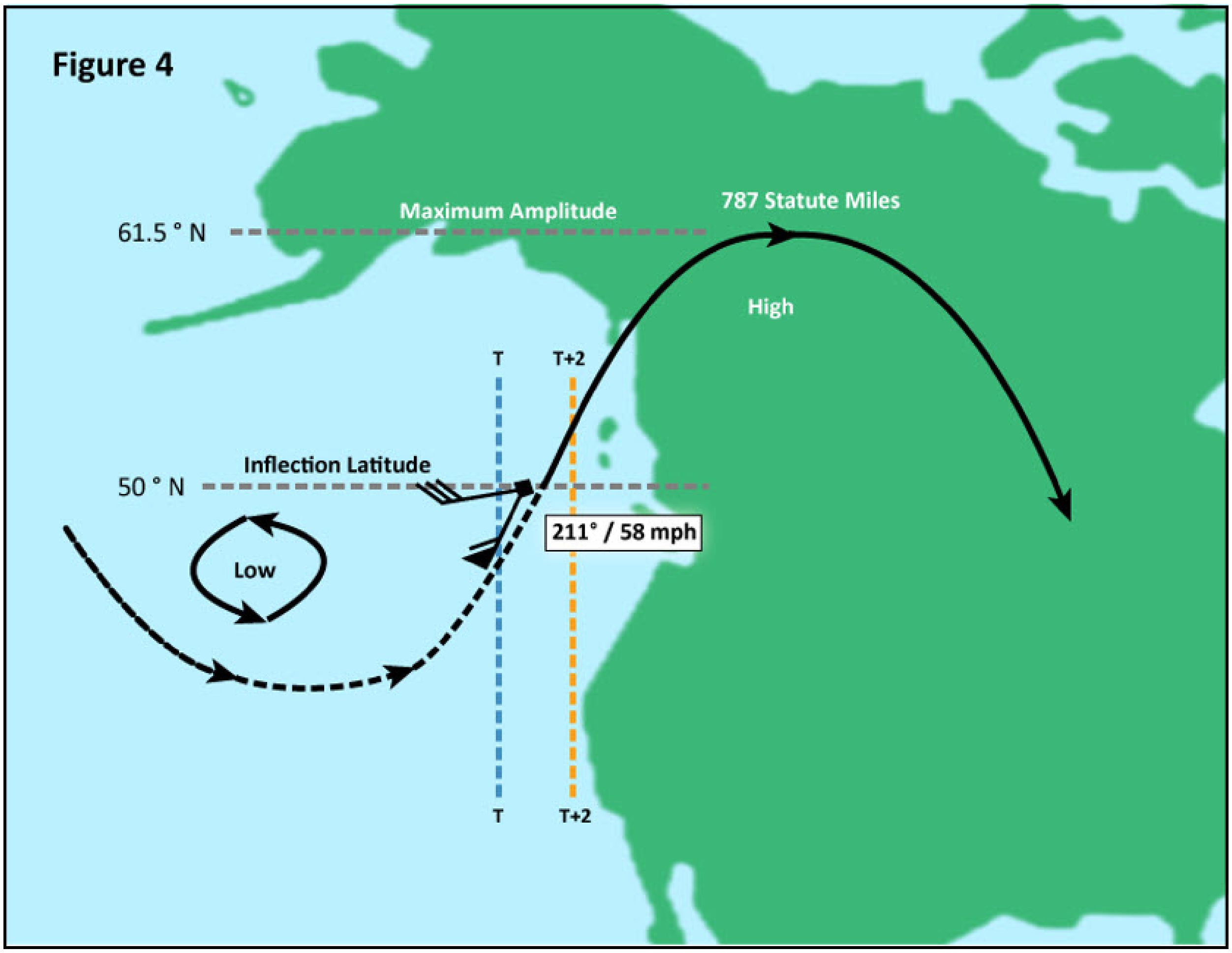
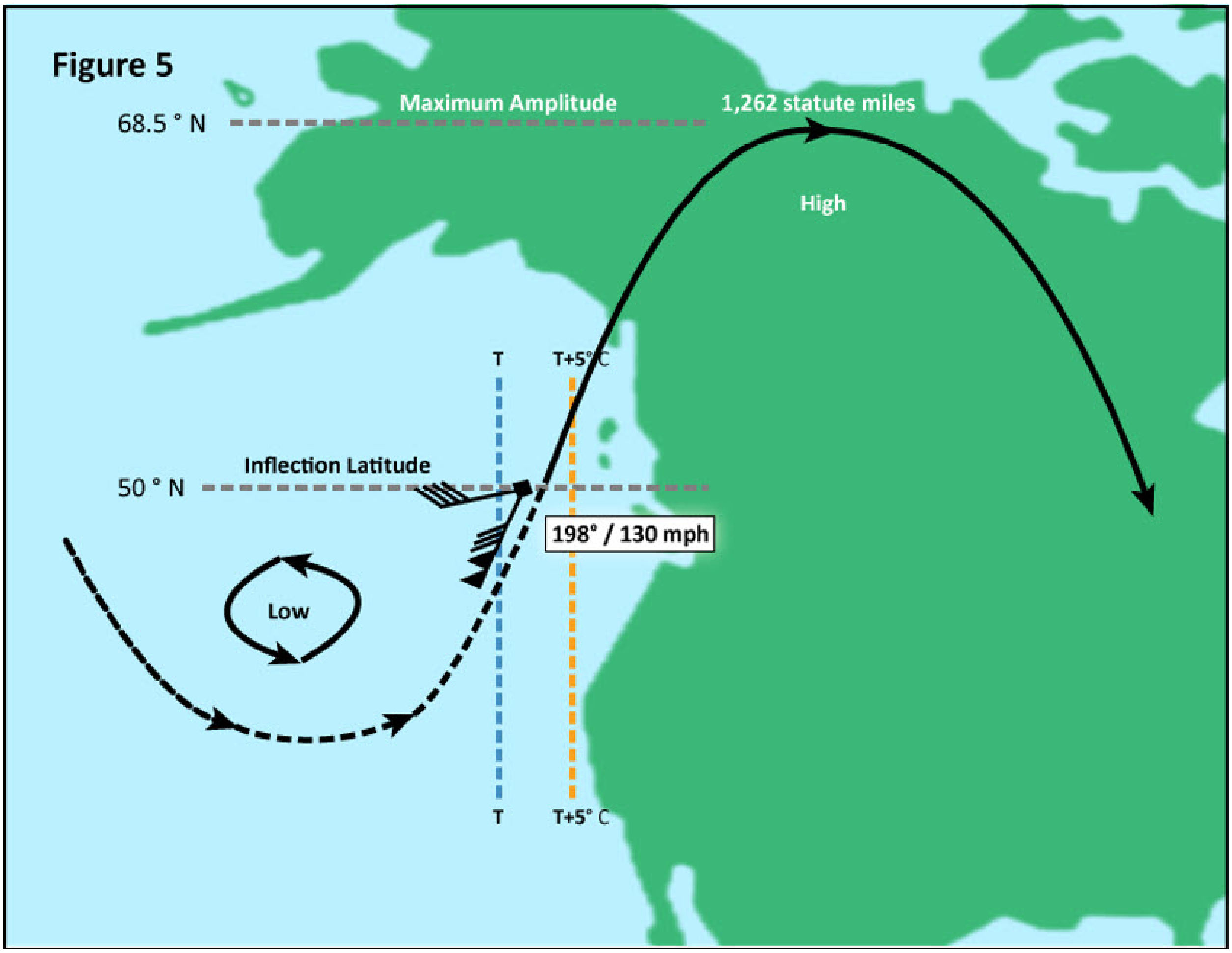


Figure 4
61.5 ° N
Maximum Amplitude
787 Statute Miles
High
T
T+2
Inflection Latitude
50 ° N
211° / 58 mph
Low
T
T+2



To further solidify these ideas, I invite the reader to examine figure 6 below. This diagram is well appreciated by meteorologists and describes the changing behavior of the atmospheres circulation as it goes through the accumulation and conversion phase of storing potential energy across the latitude lines caused by warming surplus energy from the sun at lower tropical latitudes all year that are pitted against the changing loss of this energy at the poles due to the axis of rotation of the earth tilted at 23.5 degrees with respect to the sun.

The diagram has four phases from A to D. The first phase in (A) is called the high index phase. This occurs when the latitudinal gradients of temperature are low. This phase is exactly what FV (2012) claims the earth is headed towards due to rapid warming of the Arctic caused by their claim of "Arctic Amplification".

It is called zonal flow in meteorological terminology. Notice the waves are flat and have a low amplitude as the equations tell us we would get. The speed of the jet stream is also slower in this state due to the weaker gradients of temperature across the latitude lines. Over time, and especially towards the season of winter, a much more rapid loss of energy to space by radiational cooling occurs near the polar region while the tropics continue to accumulate excess heat energy from the higher sun angles. As this temperature imbalance increases, so will the temperature gradients across the latitudes and speed of the jet stream.

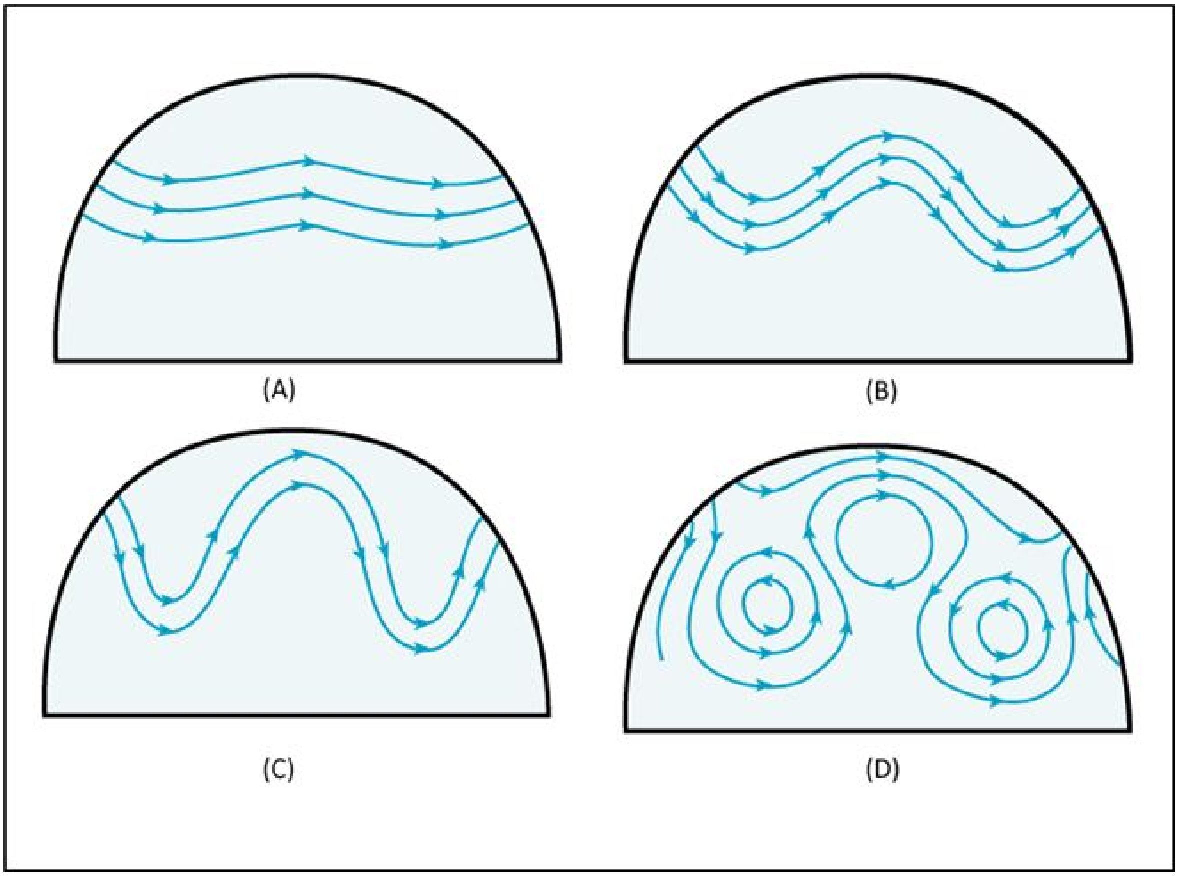
Eventually, the travelling, shorter Rossby waves in this accelerating flow have a sufficient amplitude because of reaching a critical speed for baroclinicity begin to transport heat energy poleward from the tropics. This begins the process of cyclogenesis or storm development in the troughs and we begin the change to phases (B) and (C) that are called low index flow.

Jet stream speeds in this phase continue to INCREASE, not decrease, thus increasing the amplitude of the waves as the equations tell us. This continues until we reach phase (D) where individual closed circulations of high and low pressure occur. At this point, the maximum exchange of heat energy towards the poles and cold air from the poles towards the equator occurs and the storms and high-pressure cells reach their maximum intensities and amplitudes.

Notice in this phase, just as the amplitude equation calculates, the maximum amplitudes not only occur with the higher wind speeds, but additionally from backing wind directions that exceed 90 degrees of deflection, or directions that begin to obtain an easterly component, often indicating the waves are retrogressing westward.

This process then begins the process of relaxing the latitudinal temperature gradients and from there, the Rossby wave physics begins deforming and filling the low-pressure cells and the amplitude of the flow reverses back to high index and low amplitude with an initially stronger westerly jet stream due to its displacement to a more southerly latitude. As the temperature gradients continue to weaken, so does the jet stream and it once again begins migration to higher latitudes and starts repeating the process again beginning with phase (A).

FIGURE 6



CONCLUSIONS

FV (2012) cited in the introduction of this article is fatally flawed, incorrect and should be withdrawn by the authors. As shown here, there is no theoretical basis in which to ground FV (2012). Using the proper Rossby wave physics as illustrated here, these atmospheric waves (or commonly called planetary atmospheric waves that generate low and high-pressure systems that create our weather, severe and otherwise) behave in the opposite fashion as claimed in FV (2012).

A warming Arctic that is supposed to be weakening the westerly wind belt across the northern hemisphere would create an entirely different effect on the earth's weather as FV (2012) claims. If FV (2012) claims were true, the physics governing these waves would require them to flatten in amplitude and migrate to a higher latitude, causing a much-weakened effect on the Northern Hemisphere's weather patterns.

If FV (2012) claims were true, precipitation systems would weaken and migrate northward with the migrating jet stream. Storms, severe and otherwise would become far less common than today and would be replaced with problematic drought and much higher surface absolute and relative humidities. This increased low-level moisture would lead to sporadic showers and thunderstorms in an ever-expanding maritime tropical airmass environment, but not enough precipitation to forestall severe droughts.

By severe droughts, I don't mean regional droughts such as those experienced recently in California. But rather, droughts that would expand into a worldwide regime. Present-day droughts are nothing more than cyclical changes in the earth's climate system that have very definitive and repetitive cycles.

What is particularly disturbing about FV (2012) is not only is it incorrect and flawed, but it passed peer review. Now, after publication, FV (2012) has been lapped up by media, touted and referenced in their severe weather stories that report on hurricanes, tornadoes, severe thunderstorms, heat, cold, drought and any other weather calamity as "proof" their paper is correct. Nothing could be further from the truth.

The reader needs to understand that anytime we experience severe weather, it is proof that adequate COLD in the high latitudes and Arctic has been generated by the normal radiational cooling process by the earth that creates the adequate potential energy across the latitude lines to cause amplification of the jet stream waves and speeds that pushes this colder air southward to warmer latitudes that then creates the necessary temperature gradients to liberate that energy, creating storms as well as high pressure systems.

If the occurrence of severe weather is increasing worldwide, it is not a sign of a warming earth. It is the opposite of what climate hysteria claims, and an indication of a cooling, not warming earth.

The continued misuse, abuse and general trashing of important principles founded with atmospheric science remains as deplorable as ever by the groups promoting global warming from human CO2 emissions or by these same groups promoting climate hysteria by re-labeling this term "climate change".

Now that the flawed FV (2012) passed peer review, it allows media to blame any severe weather on "climate change." FV (2012) allows media to claim a wavier jet stream dips and meanders because the Arctic is supposedly getting warmer. All this is sheer nonsense and all demonstrably wrong.

I believe this flawed FV (2012) also shows how the quality of the scientific peer review process has been lowered in "climate science".

APPENDIX 1

DERIVATIONS OF THE APPROPRIATE EQUATIONS FOR THIS ARTICLE

Large atmospheric waves as analyzed and seen on synoptic weather maps behave according to their derivation from the atmospheric vorticity theorem. Vorticity is another term used in atmospheric dynamics that describes spin motion characteristics in a stream flow of air such as the jet stream. The spinning behavior of the air in such a flow is generated by the spherical geometry of the earth and the earth's rotation by itself as well as speed shearing along these rivers of air that surround the earth at higher altitudes.

This introduces terms such as the Coriolis force and the Rossby parameter, each assuring because of the earth's rotation that the absolute vorticity of the earth be conserved across the lines of latitude from equator to pole, or defined as

$d(f + \zeta)/dt = 0$

where f is the Coriolis parameter, defined as

$f = 2\,\Omega \sin \phi$

where,

Ω = the angular speed of the earth = $7.292\times10^{-5}\ s^{-1}$

ϕ = the selected earth latitude

ζ = the relative vorticity about the vertical axis at any point on the earth and is defined as

$\zeta = \partial v/\partial x - \partial u/\partial y$

where u and v are the respective west to east and south to north wind components.

So the time derivative of the sum of f and ζ being equal to zero means that if the relative vorticity about the vertical increases at a point, the Coriolis parameter must decrease an equal amount, which as we see is latitude dependent, so the rate of change of absolute vorticity at any point across the latitude lines is conserved and always zero but changes relative to the latitude, which defines the relative vorticity.

In the derivation of the Rossby wave equation, it should be noted that we need to use the rate of change of the Coriolis parameter, f, so we take the derivative of f with respect to ϕ, then in Cartesian coordinates,

$df/d\phi = 2\,\Omega \cos(\phi/a) = df/dy$

where

a = the radius of the earth.

Note that y represents the north-south axis which represents the changing lines of latitude from equator to pole and the derivative of f is then divided by a which is the mean radius of the earth.

This gives us the change in the latitudinal dependent relative vorticity about the vertical in dimensions of $m^{-1}\ s^{-1}$ because we divided by a.

An important physical characteristic of what we have done so far is to make it clear that the relative vorticity about the vertical increases with decreasing latitude. This has a significant meaning to the development of long waves and storm systems in the westerlies in that any wave cyclone or low pressure system that propagates along such a wave is subject to a "spin up" or intensification if it moves to a lower latitude and gains relative vorticity about the vertical. Likewise, northward moving systems lose relative vorticity to Coriolis turning and are subject to spin down or weakening.

APPENDIX 2

Now we turn to the construction of the Rossby wave equation used in atmospheric science to check on the validity of the claims made by FV (2012). Assumptions need to be made to this construction that simplify the mathematics considerably because we find if it is derived from the vorticity equation it winds up being a second order, nonlinear partial differential equation with a product of dependent variables because of the separate *u* and *v* wind components and Rossby parameter.

It is simplified considerably by making some assumptions that are actually beneficial to what meteorologists are interested in knowing about the behavior of these waves as they occur in the earth's atmospheric system. And because that is very large, we can eliminate some of the cross dependence in the equation by making the individual derivatives follow a point in the stream rather than a parcel of air directly so that the waves have a constant shape and follow a large river of air around the hemispheres that we actually see and define as the westerlies that encircle both hemispheres.

That large river of air is then taken as *U* rather than a localized *u* component and disturbances in the flow are perturbed by introducing a *v* component, *v'* of velocity into the flow that is a function of the *x-axis* (west to east) and time, *t*. As stated above, absolute vorticity on the earth is conserved so that from Hess (1, 16.4, 16.5)

$$d(f+\zeta)/dt \equiv \frac{\partial \zeta}{\partial t} + u\frac{\partial \zeta}{\partial x} + v\frac{\partial \zeta}{\partial y} + v\frac{\partial f}{\partial y} = 0 \qquad (1)$$

If we describe the wind components to consist of a broad westerly wind current with a north/south wave pattern of infinite lateral extent, then the dependent variable can become independent of *y* so that we have a system of waves in which the streamlines at any latitude are parallel to any other latitude. Then

$$\zeta = \frac{\partial v}{\partial x}$$

Giving

$$\frac{\partial v}{\partial x}\frac{\partial}{\partial t} + u\frac{\partial^2 v}{\partial x^2} + \beta v = 0 \quad (2)$$

If one were to follow a point moving in the west-east direction with speed *c*, no changes will be observed in any of the variables. That is the operator,

$$D/Dt = \frac{\partial}{\partial t} + c(\frac{\partial}{\partial x}) = 0 \qquad (3)$$

D/Dt is an individual derivative following a point moving with speed *c*. This is different than the derivative d/d*t* that would follow a parcel of air directly. With these assertions, the vorticity equation becomes,

$$(u - c)\frac{\partial^2 v}{\partial x^2} + \beta v = 0 \qquad (4)$$

This is a difficult to solve, nonlinear equation with a product of dependent variables. To simplify it we assume the above conditions describing *u* as a large river of west to east moving air *U* encompassing the entire hemisphere, with superimposed and smaller perturbations of *u'* and *v'* travelling within in it that are functions of *x* and *t.* The nomenclature is then,

$$u(x,t) = U + u'(x,t)$$
$$v(x,t) = v'(x,t) \qquad (5)$$

Then U becomes the zero order of magnitude because it is large compared to *u'* or *v'* as we choose to make those planetary waves about 1/10 as large as *U*. So, the perturbations are then a first order of magnitude to *U* as they are an order of 1/10 on a logarithmic scale, and the additional resulting terms that follow from this rearrangement are an order of magnitude smaller yet, or 1/100 of *U* on a logarithmic scale and because of this are of a second order of magnitude. We then end up with the following linear second order partial differential equation with constant coefficients,

$$(U - c)\frac{\partial^2 v'}{\partial x^2} + \beta v' + u'\frac{\partial^2 v'}{\partial x^2} = 0 \qquad (6)$$

With the above stated analogy, the first two terms are then a first order of magnitude to *U* but the third is a product of the first order magnitudes making it a second order of magnitude or 1/100 as large as *U*. The result of this is that the third term is sufficiently small to ignore. We can now simplify this differential equation to

$$\partial^2 v'/\partial x^2 + \beta v'/U - c = 0 \qquad (7)$$

Where *beta* is the Rossby parameter we have already defined and the newer term *c* is the speed of the waves. A general solution to this equation as we have defined the functions is trigonometric and we have a solution with v_{0x} representing the maximum wave amplitude from trough to ridge axis. Then

$$v' = v_{0x} sin(2\pi x/L) \qquad (8)$$

Substituting for *v'*

$$\frac{\partial^2}{\partial x^2} v_{0x} sin(2\pi x/L) + \beta v_{0x} sin(2\pi x/L)/U - c = 0 \qquad (9)$$

Thus

$$\beta v_{0x} sin(2\pi x/L) = 4\pi^2/L^2 [v_{0x} sin(2\pi x/L)][U - c] \qquad (10)$$

Which then reduces the differential equation to

$$U - c = \beta L^2/4\pi^2 \qquad (11)$$

This is the Rossby equation that was derived in 1939 by Carl Rossby that describes the frequency of atmospheric waves by their lengths that become a function of their speeds from both the general speed of the westerly jet stream surrounding the earth and the speeds of the smaller waves themselves that traverse within the larger river of air called the westerlies.

One of the disadvantages of this equation is that it does not speak directly to the amplitude of these waves even though the maximum amplitude is specified in the general solution. That must be assumed a constant in the formulations described and vanishes as such in the final solution. I want to address this part of the problem with much greater specificity, so I will introduce more dynamics for a definition of the wave amplitudes.

APPENDIX 3

As the Rossby waves are introduced in the prior solutions, the amplitudes like the wavelengths themselves must be a function of Coriolis turning as defined above. Martin (2, 3) notes this in writing the expression that the ratios of wind velocity along a streamline to radius of curvature of the flow is described by

$$V/R = -f + f_1 + V/R_1 \qquad (12)$$

Where *V* and *R* are the wind speed and radius of curvature of the flow respectively and *f* and f_1 is the Coriolis parameter defined at an initial point along *R* and a nearby point R_1, respectively.

$$f - f_1 = 2\Omega(sin\rho - sin\rho_1) \qquad (13)$$

Which describes the magnitude of Coriolis turning and the prescribed latitudes ρ and ρ_1 that affect it. Ω has already been defined as the angular velocity of the earth. $f - f_1$ is also equivalent to the Rossby parameter and product of latitudinal displacement which may be written as

$$f - f_1 = 2\Omega cos\rho_1/a(y - y_1) = \beta(y - y_1) = df/dy \qquad (14)$$

Recalling and defined previously that the Rossby parameter is df/dy. The new variables are now y and y_1 that define the latitudinal displacement of the wave along a particular meridian which defines the amplitude along the meridian or axis in Cartesian coordinates. The objective in quantifying the amplitude of Rossby waves is also to make the "flat earth" Cartesian coordinates to a spherical form that represents the true latitudinal distances of the earth. Platzman (1947) had achieved this by showing that if a is the radius of the earth as earlier defined, then

$$1/R = -1/a[d(cos\psi cos\rho)/d(sin\rho)] \quad (15)$$

Where,

psi is the wind direction measured relative to a latitude circle on Lambert conformal or polar stereographic coordinates. Combining (12) and (15) yields

$$d(cos\psi cos\rho)/d(sin\rho) = (f - f_1)a/V - a/R_1 \qquad (16)$$

Then

$$\int_{\rho_1}^{\rho} d(cos\psi cos\rho) = 2\Omega \int_{\rho_1}^{\rho} d(sin\rho)(sin\rho - sin\rho_1)a/V - \int_{\rho_1}^{\rho} d(sin\rho)a/R_1$$

Resulting in

$$cos\psi cos\rho = cos\psi_1 cos\rho_1 + \Omega a/V(sin\rho - sin\rho_1)^2 - a/R_1(sin\rho - sin\rho_1)$$

The first order approximation is,

$$sin\rho - sin\rho_1 = cos\rho_1(y - y_1)/a$$

Consequently

$$cos\psi = sec\rho cos\rho_1[cos\psi_1 + \beta/2V(y - y_1)^2 - (y - y_1)/R_1] \quad (17)$$

Equation (17) is useful now because it can be further resolved in aiding the construction of a Rossby wave with the appropriate data. Namely, this equation can be further resolved to provide a definition of inflectional latitude and inflectional wind direction as defined by Martin (3) providing an inflectional streamline exists, which by definition will exist providing that,

$$4R_1^2 cos\psi_1 > 2V/\beta \qquad (18)$$

This would tell us that the cyclonic curvature radius on the left term must be larger than the effects of Coriolis turning on the right-hand side. In most synoptic scale systems, we find this is almost always the case. But we are not interested in the actual construction of the complete wave, but rather, we want an accurate assessment of the wave amplitude regardless of the meridian that the ridge or trough axis positions itself on. The point of this article is to tie in wave amplitude with wind speed and wind speed to the claimed weakening latitudinal temperature gradients and cross-check those computations with the claims made in FV (2012).

To do this, we can simplify (17) further by taking the arbitrary initial point as the inflection point or latitude which allows us to set the third term in brackets on the right-hand side of (17) to zero. We would then be computing the wave amplitude from the inflection latitude point rather than from the full trough axis that involves. This would then be approximately one-half of a full trough to ridge amplitude. This is perfectly acceptable because the ridge axis is what these authors claim is expanding northward due to AA or Arctic Amplification warming. They claim this process is “stretching” the waves in amplitude and stalling them out leading to extreme and persistent weather events.

In reality, we would find that deepening or intensifying troughs most always carry higher jet speed strength as well that takes us to the inflection latitude to compute the wave amplitude from that point. So the trough axis often expands to a lower latitude from deepening as does the northward stretching or shrinking from wind speed along the streamlines. Following this procedure, equation (17) simplifies further and can be solved for $y - y_1$.

The result is,

$$v_0 = y - y_0 = 2\sqrt{V/\beta[(cos\psi/sec\rho cos\rho_1) - cos\psi_1)]} \qquad (19)$$

This is the equation we want to cross check the Rossby wave amplitude. As a reminder that this equation does not give us a full trough to ridge amplitude, the subscripts v_0 and y_0 are introduced to define the amplitude from the inflection latitude respectively, not v_0 as was used in the Rossby wavelength amplitude that defines the maximum amplitude selected from trough to ridge axis.

Likewise, the initial points rho_1 and psi_1 now become the inflectional latitude and inflectional wind direction respectively and ψ and ρ are the wind direction at the ridge axis and ridge axis latitude respectively. At the ridge axis, the wind direction would always be from the west so that *psi* = 0.

Then any arbitrary inflectional latitude, wind direction and speed can be chosen.

In this article, I selected 50 degrees North latitude but the wind direction and speed at this point was computed by changing the temperature gradients across an arbitrary frontal boundary to increase or decrease the directions and speeds accordingly to tie together the effects of temperature gradients, wind speed, direction and Rossby wavelength and amplitude to verify or nullify the claims made in FV (2012).

The set of equations to do this are now complete. However, I advise caution when attempting to use this equation in calculating low latitude wave amplitudes. Below 40 degrees north latitude, the importance of radius of curvature begins to dominate Coriolis turning and can result in higher wave amplitude projections resulting in a greater margin of error compared to the published actual CAVT trajectories.

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