

Derivation of equations and example calculations of the components of CO₂

by

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Introduction

The methodology we use to estimate the annual mean values for the anthropogenic-fossil component, $C_F(t)$, and the non-fossil component, $C_{NF}(t)$, requires starting at the end of 1750, the assumed onset of the industrial revolution and burning of fossil fuels. The annual mean total CO₂ concentration in 1750, $C(0)$, and its ¹⁴C specific activity, $S(0)$, are compared to values $C(t)$ and $S(t)$ in calculating annual mean values for the two components at t years after 1750. By definition, no anthropogenic-fossil CO₂ is present in 1750 in $C(0)$. Although $C(0)$ in 1750 has an unspecified, natural background fossil component, such as emissions from volcanos, it is small in comparison to the anthropogenic-fossil component, $C_F(t)$, and this natural background fossil component is not considered directly in the derivation of the equation for $C_F(t)$. Thus, $C(0)$ is considered not to have anthropogenic-fossil CO₂, and it also is designated by the symbol $C_{NF}(0)$. The anthropogenic-fossil emissions of CO₂ increased in 1950 to 2010 from 30 times to 135 times that from volcanos, which are the major source of natural fossil CO₂.

Equations in either of two logic pathways can be used to estimate the two components of CO₂. Our paper uses equations in Pathway 2, which provides a more concise but complete presentation of the basis of our methodology and calculation of all CO₂ quantities of interest. The derivation of equations for Pathway 1 and example calculations below are used to provide additional clarification for readers of our paper titled, “World atmospheric CO₂, its ¹⁴C specific activity, non fossil component, anthropogenic-fossil component, and emissions (1750 - 2018)”. It is assumed that the reader of this document has an understanding of the definitions of all symbols used in our paper. The derivation of the equations for annual mean CO₂ quantities in Pathway 1 below provides a more detailed derivation of all CO₂ quantities than that in our paper. Both pathways and their equations are summarized in this document later. Example calculations below are given for annual mean CO₂ quantities in 2018, which is 268 y after 1750 and start of the industrial revolution. These calculated quantities are compared to values in the attached Table 2a calculated by equations in Pathway 2.

Discussion

Major assumptions used to derive equations for CO₂ annual mean quantities

The major assumptions used to derive the equations for the CO₂ annual mean quantities present each year in the atmosphere are summarized as follows. The total CO₂ concentration, $C(t)$, has had an essentially constant value of 280 ppm for several thousand years prior to 1750. It is assumed that the production rate of ¹⁴C in the atmosphere from the interaction of cosmic rays has been essentially constant for at least fifteen thousand years, and the world total activity of ¹⁴C has maintained a steady state value. Except for the variation of the annual change, $DC_{NF}(t)$, in the concentration of the non fossil component from one year to the next due to the redistribution of CO₂ in its reservoirs and changes in land use, the activity of ¹⁴C per unit volume of the atmosphere then would be constant. The initial ¹⁴C activity per unit volume of air in 1750 is proportional to $C(0)S(0)$. The product of $C(t)S(t)$ is directly proportional to the activity of ¹⁴C per unit volume of air at time t years after 1750. Therefore, the activity per unit volume of air would be constant except for the annual increase or decrease of the product of $DC_{NF}(t)S(t)$. The $S(t)$ value equals the specific activity of ¹⁴C in $DC_{NF}(t)$ as well as in $C(t)$ because all three CO₂ quantities represent annual mean values in the same year. The annual change in the concentration of the anthropogenic-fossil component, $DC_F(t)$, from the value of zero in 1750 is the same as its annual mean value, $C_F(t)$, present at time t years after 1750. The annual change, $DC(t)$ in $C(t)$ is given by the sum: $DC(t) = DC_{NF}(t) + DC_F(t)$. The annual mean total concentration, $C(t)$, present at time t can be expressed: $C(t) = C(0) + DC_{NF}(t) + DC_F(t)$ or by: $C(t) = C_{NF}(t) + C_F(t)$.

Derivation of equations for Pathway 1

The major assumptions above are used for the derivation of the equation for the $DC_{NF}(t)$ value in 2018 in relation to the initial non fossil concentration, $C_{NF}(0)$, i.e., $C(0)$ of 276.44 ppm in 1750. The sign of $DC_{NF}(t)$ can be positive or negative depending respectively on whether or not there is a net amount of CO_2 into or out of the atmosphere in any year since 1750. The annual mean specific activities, $S(t)$, that are calculated from the annual mean $D^{14}C$ values for the atmosphere each year represent the annual mean specific activities present that year in the atmosphere. The initial non fossil concentration, $C(0)$, of 276.44 ppm and its specific activity, $S(0)$, of $16.33 \text{ dpm}(\text{gC})^{-1}$ in 1750 ($t = 0$) listed in the attached Table 2a are considered as annual mean values before the start of the industrial revolution after 1750. In 2018 ($t = 268 \text{ y}$), the total CO_2 concentration, $C(268 \text{ y})$, is 405.40 ppm, and its expected specific activity, $\langle S(268 \text{ y}) \rangle$, is $13.96 \text{ dpm}(\text{gC})^{-1}$. These values are used below to calculate values for other CO_2 quantities in 2018 by the equations for Pathway 1. Values of CO_2 quantities listed in Table 2a were calculated by the equations in Pathway 2, and they would be the same as those for Pathway 1 except for roundoff errors. For ease of reference, equation numbers listed below are the same as the numbers listed for each section below and not the equation numbers listed at the head of columns of Table 2a.

- (1) The statement for the annual change at time t of the ^{14}C activity per unit volume of air from that in 1750 is expressed based on a conversion constant K in units of $\text{gCm}^{-3}(\text{ppm})^{-1}$:

$$KDC_{NF}(t)\langle S(t) \rangle = KC(t)\langle S(t) \rangle - KC(0)S(0), \text{ which yields:}$$

$$DC_{NF}(t) = \left(\frac{C(t)\langle S(t) \rangle - C(0)S(0)}{\langle S(t) \rangle} \right) = \left(\frac{(405.40)(13.96) - (276.44)(16.33)}{(13.96)} \right) \text{ ppm} = 82.03 \text{ ppm}, \quad (1)$$

which differs slightly from the value of 82.12 ppm in Table 2a because not all of the digits in the value of $\langle S(t) \rangle$ in Table 2a have been used in the calculation above. When $C(t)\langle S(t) \rangle$ exceeds $C(0)S(0)$, then $DC_{NF}(t)$ is positive. When $C(t)\langle S(t) \rangle$ is less than $C(0)S(0)$, then $DC_{NF}(t)$ is negative. When $C(t)\langle S(t) \rangle$ equals $C(0)S(0)$, then $DC_{NF}(t)$ is zero, and $C_{NF}(t)$ equals $C(0)$. The specific activity then is given by: $\langle S(t) \rangle = C(0)S(0)C(t)^{-1} = C(0)S(0)[C(0) + C_F(t)]^{-1}$, which as demonstrated later applies regardless of the sign or magnitude of $DC_{NF}(t)$. In this equation and others below, $\langle S(t) \rangle \leq S(0)$ because the anthropogenic-fossil component, $C_F(t)$, increases each year after 1750. If a value greater than $C(0)$ or $S(0)$ had been chosen, then the calculated value of 82.03 ppm for $DC_{NF}(t)$ would have been less and the anthropogenic-fossil component, $C_F(t)$, would have been more. The value of $C_F(t)$ is not included in the equation for $DC_{NF}(t)$ because its presence in the atmosphere does not add any activity of ^{14}C per unit volume of the atmosphere.

- (2) The 2018 non fossil concentration, $C_{NF}(t)$, at time t of 268 years is calculated:

$$C_{NF}(t) = C(0) + DC_{NF}(t) = (276.44 + 82.03) \text{ ppm} = 358.47 \text{ ppm}, \quad (2)$$

which differs slightly from the value of 358.56 ppm in Table 2a calculated by equation in Pathway 2.

- (3) The 2018 anthropogenic-fossil concentration, $C_F(t)$, at time t of 268 years is calculated:

$$C_F(t) = C(t) - C_{NF}(t) = (405.40 - 358.47) \text{ ppm} = 46.93 \text{ ppm}, \quad (3)$$

which differs slightly from the value of 46.84 ppm in Table 2a calculated by equation in Pathway 2.

(4) The 2018 fraction, $f_F(t)$, of anthropogenic-fossil CO₂ in $C(t)$ at time t of 268 y is calculated:

$$f_F(t) \equiv \frac{C_F(t)}{C(t)} = \frac{46.84}{405.40} = 0.116 \text{ or } 11.6\%. \quad (4)$$

(5) The 2018 fraction, $f_{NF}(t)$, of non fossil CO₂ in $C(t)$ at time t of 268 y is calculated:

$$f_{NF}(t) \equiv \frac{C_{NF}(t)}{C(t)} = 1 - f_F(t) = 1 - 0.116 = 0.884, \text{ or } 88.4\%. \quad (5)$$

(6) To provide further insight about the relationships among quantities underlying the first three equations above, they are derived or expressed as follows. The algebraic expression for $DC_{NF}(t)$ in Equation (1) is key to the derivation and calculation of all quantities listed in the equations. The expression for $DC_{NF}(t)$ is derived alternatively using a thought experiment. Assume all of the carbon in the samples used to obtain the annual mean concentration $C(t)$ of 405.40 ppm in 2018 is later used to measure its 2018 specific activity $\langle S(t) \rangle$ of 13.96 dpm(gC)⁻¹. Assume that the volume of all the air samples is V cubic meters. Based on a constant K in the units of gCm⁻³(ppm)⁻¹, the following relationships can be established for the expected specific activity, $\langle S(t) \rangle$:

$$\langle S(t) \rangle = \left(\frac{KVC(0)S(0) + KVDC_{NF}(t)\langle S(t) \rangle}{KVC(t)} \right) = \left(\frac{C(0)S(0) + DC_{NF}(t)\langle S(t) \rangle}{C(t)} \right), \text{ thus:} \quad (6)$$

$$DC_{NF}(t) = \left(\frac{C(t)\langle S(t) \rangle - C(0)S(0)}{\langle S(t) \rangle} \right),$$

which is the same expression as that in Equation (1). The terms in the first equality represent:

$KVC(0)S(0) \equiv$ activity of ¹⁴C in air volume V in 1750;

$KVDC_{NF}(t)\langle S(t) \rangle \equiv$ activity of ¹⁴C lost or added to air volume V in 2018.

$KVC(t) \equiv$ total mass in grams of carbon in the air volume V in 2018.

(7) An expression for the expected specific activity, $\langle S(t) \rangle$, can be obtained from the last expression in Equation (6), which yields the value of 13.96 dpm(gC)⁻¹ for $\langle S(268 \text{ y}) \rangle$ in Table 2a:

$$\langle S(t) \rangle = \frac{C(0)S(0)}{C(t) - DC_{NF}(t)} = \frac{(276.44 \text{ ppm})(16.33 \text{ dpm(gC)}^{-1})}{405.40 \text{ ppm} - 82.03 \text{ ppm}} = 13.96 \text{ dpm(gC)}^{-1}. \quad (7)$$

(8) It can be shown that the algebraic expression in Equation (7) for the expected specific activity, $\langle S(t) \rangle$, can be reduced to an expression which also yields the value, $\langle S(268 \text{ y}) \rangle$ of 13.96 dpm(gC)⁻¹:

$$\langle S(t) \rangle = \frac{C(0)S(0)}{C(t) - DC_{NF}(t)} = \frac{C(0)S(0)}{C(t) - [C_{NF}(t) - C(0)]} = \frac{C(0)S(0)}{C(t) - [(C(t) - C_F(t)) - C(0)]}, \text{ or} \quad (8)$$

$$\langle S(t) \rangle = \frac{C(0)S(0)}{C(0) + C_F(t)} = \frac{(276.44 \text{ ppm})(16.33 \text{ dpm(gC)}^{-1})}{276.44 \text{ ppm} + 46.84 \text{ ppm}} = 13.96 \text{ dpm(gC)}^{-1}.$$

- (9) When the expression for $DC_{NF}(t)$ in Equation (1) is substituted in Equation (2), the non fossil component, $C_{NF}(t)$, also can be expressed:

$$C_{NF}(t) = C(0) + \left(\frac{C(t)\langle S(t) \rangle - C(0)S(0)}{\langle S(t) \rangle} \right) = C(t) - \left(\frac{S(0)}{\langle S(t) \rangle} - 1 \right) C(0), \text{ for } \langle S(t) \rangle \leq S(0), \quad (9)$$

which indicates that $C_{NF}(t)$ is larger for either larger values of $C(t)$ or of $\langle S(t) \rangle$. When $\langle S(t) \rangle$ equals $S(0)$, then $C_{NF}(t)$ equals $C(t)$. If a value larger than either the chosen value of $C(0)$ of 276.44 ppm or of the chosen value of $S(0)$ of 16.33 dpm(gC)⁻¹ is selected later, then the calculated value of $C_{NF}(t)$ will be less, and the fossil component, $C_F(t)$, will be more.

- (10) By Equation (8), the expected anthropogenic-fossil component, $\langle C_F(t) \rangle$, can be expressed:

$$\langle C_F(t) \rangle = \frac{C(0)S(0)}{\langle S(t) \rangle} - C(0) = \left(\frac{S(0)}{\langle S(t) \rangle} - 1 \right) C(0), \text{ for } \langle S(t) \rangle \leq S(0), \quad (10)$$

which equals zero when $\langle S(t) \rangle$ equals $S(0)$ and increases as $\langle S(t) \rangle$ decreases from one year to the next. The expected anthropogenic-fossil component, $\langle C_F(t) \rangle$, is independent of the total concentration, $C(t)$. It varies each year only with the expected specific activity, $\langle S(t) \rangle$, of the total concentration, $C(t)$. It is noted that this expression for calculating $\langle C_F(t) \rangle$ could have been the starting point for calculating all other quantities of interest as indicated by the equations in Pathway 2 shown later. Equation (10) only requires knowledge of the initial concentration, $C(0)$, in 1750 and the specific activities, $S(0)$ and $\langle S(t) \rangle$, of ¹⁴C in, respectively, $C(0)$ and $C(t)$. It is noted that a smaller estimate of either $S(0)$ or $C(0)$ would yield smaller values of $\langle C_F(t) \rangle$ and larger values of $C_{NF}(t)$.

Summary and Conclusions

- (1) All calculated CO₂ quantities represent annual mean values present at time t after 1750 and/or annual changes from values in 1750, and they automatically account for the redistribution of all isotopic forms of CO₂ among its reservoirs and changes in land use.
- (2) The product $C(0)S(0)$ is directly proportional to the ¹⁴C activity per unit volume of air in 1750. In subsequent years, the activity per unit volume is proportional to $C(t)S(t)$ regardless of the amount, $C_F(t)$, of anthropogenic-fossil CO₂ but varying with the value of the annual change, $DC_{NF}(t)$, of the non-fossil component each year.
- (3) The annual change, $DC_F(t)$, in the anthropogenic-fossil component from the value of zero in 1750 is the same as the amount present, $C_F(t)$, which depends only on the ¹⁴C activity per gram of carbon, i.e., the specific activity, $S(t)$ of $C(t)$, and not $C(t)$ itself or $DC_{NF}(t)$.
- (4) The annual changes in the non fossil component, $DC_{NF}(t)$, and anthropogenic-fossil component, $DC_F(t)$, are independent processes whose sum is the annual change, $DC(t)$, in $C(t)$.
- (5) As indicated in our paper, the initial value of $C(0)$ of CO₂ and its specific activity, $S(0)$, may be somewhat overestimated. Consequently, all values of the anthropogenic-fossil component, $C_F(t)$, may be overestimated, and values of the non-fossil component may be under estimated.
- (6) The values of all CO₂ quantities of interest can be calculated by either the equations in Pathway 1 or Pathway 2, and they would have the same values except for roundoff of digits in values of quantities.
- (7) Pathway 1 begins with the equation for calculating the annual change, $DC_{NF}(t)$, in the non-fossil component from its initial value, $C_{NF}(0)$, i.e., $C(0)$, of 276.44 ppm in 1750. Pathway 2 begins with the equation for calculating the value of the anthropogenic-fossil component, $C_F(t)$, present at time t relative to its initial value of zero in 1750. The two pathways and their equations are summarized in (8) and (9) below.

(8) Pathway 1 is expressed when $S(t)$ is replaced by $\langle S(t) \rangle$ in the paper:

$$DC_{NF}(t) = [C(t) \langle S(t) \rangle - C(0) S(0)] \langle S(t) \rangle^{-1} \rightarrow C_{NF}(t) = C(0) + DC_{NF}(t) \rightarrow C_F(t) = C(t) - C_{NF}(t).$$

(9) Pathway (2) is expressed when $S(t)$ is replaced by $\langle S(t) \rangle$ in the paper:

$$\langle C_F(t) \rangle = [S(0) \langle S(t) \rangle^{-1} - 1] C(0) \rightarrow C_{NF}(t) = C(t) - \langle C_F(t) \rangle \rightarrow DC_{NF}(t) = C_{NF}(t) - C(0).$$

(10) Results for the CO₂ components in Table 2a below negate the claims of the dominance of the anthropogenic component, $C_F(t)$, in the annual increases, $DC(t)$, of the total concentration, $C(t)$, that began in about 1800. Yet, significant emissions, $DE(t)$, of anthropogenic fossil CO₂ did not start until 1900, which is 100 years after the start of the increase in the total concentration, $C(t)$. Most of the annual increase, $DC(t)$, in $C(t)$, is due to the annual increase, $DC_{NF}(t)$, in the value of the non-fossil component, $C_{NF}(t)$, which is due the annual redistribution of all CO₂ components among its reservoirs. As indicated in the paper, most of this increase in $DC_{NF}(t)$ is due to a net input to the atmosphere from the oceans due to its heating from a solar heating cycle that continues today.

Table 2a. Partial world atmospheric CO₂, its ¹⁴C specific activity, anthropogenic fossil and non fossil components, and emissions (1750 - 2018)

Year	Time t	$C(t)$	(2)	(3)	(4)	(5)	(6)	(7)
	since 1750							
	(years)	(ppm)	(dpm (gC) ⁻¹)	(ppm)	(ppm)	(ppm)	(ppm)	(bill. met. tons)
1750	0	276.44	16.33	0.00	276.44	0.00	0.00	0.00
1751	1	276.40	16.33	0.01	276.39	-0.05	-0.04	0.01
1800	50	281.86	16.31	0.42	281.44	5.00	5.42	0.03
1850	100	287.06	16.28	0.84	286.22	9.78	10.62	0.20
1900	150	296.03	16.25	1.40	294.63	18.19	19.59	1.96
1950	200	311.54	16.10	4.03	307.51	31.07	35.10	5.83
2000	250	367.43	15.06	23.39	344.04	67.60	90.99	24.53
2018	268	405.40	13.96	46.84	358.56	82.12	128.96	36.22

Note: The sum of $DE(t)$ values each year through 2018 is 1.590×10^{18} g. In 2018, the $\langle C_F(t) \rangle$ value of 46.84 ppm present in the atmosphere corresponds to a total mass estimated as 3.664×10^{17} g or about 23% of the total emissions. Thus in 2018, 77% of the total emissions is in the atmosphere's exchange reservoirs (See text in paper.)

(11) The assumption that the increase in CO₂ since 1800 is dominated by or equal to the increase in the anthropogenic component is not settled science. Unsupported conclusions of the dominance of the anthropogenic-fossil component of CO₂ and concerns of its effect on climate change and global warming have severe potential societal implications that press the need for very costly remedial actions that are presently unnecessary, ineffective in curbing global warming, and misdirected to such extent that they threaten the very existence of life on earth.

Other Support of the Above Conclusions

Video: Conversations on Climate Change

On June 12, 2018 renowned experts, Dr. Michael Mann, Dr. David Titley, Dr. Patrick Moore and Dr. Judith Curry met in Charleston, West Virginia to discuss climate change from varying perspectives. The panelists were asked to address two specific questions: To what extent is the use of fossil fuels affecting climate change? What can and should be done to offset those effects? This event was presented by Spilman Thomas & Battle, a full-service, super-regional law firm serving local, regional, national and international businesses. ©2018 Spilman Thomas & Battle, PLLC. It is important to realize that the four speakers represent different opinions as indicated by the four short video clips of each participant's presentation prior to their full presentations. Dr. Patrick Moore, the last speaker, has opinions and presentations of data that strongly oppose those of Dr. Michael Mann and Dr. David Titley. The link for the video is:

<https://www.youtube.com/watch?v=lyNCI7NzjaM>

Article by Patrick Moore

The following article contains data presented by Patrick Moore in the above video.

**THE POSITIVE IMPACT OF HUMAN CO₂ EMISSIONS
ON THE SURVIVAL OF LIFE ON EARTH
BY PATRICK MOORE | JUNE 2016**

The link for the article is:

<https://fcpp.org/wp-content/uploads/2016/06/Moore-Positive-Impact-of-Human-CO2-Emissions.pdf>

We believe that both the video and article of Patrick Moore support our conclusions above, especially and most importantly, conclusion (11). Efforts by Patrick Moore to prevent misdirected actions such as limiting the burning of fossil fuels and sequestering CO₂ before it is released to the atmosphere deserve our consideration.